Texture Analysis and Segmentation using Modulation Models

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Image Processing Seminar

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Presentation Outline

Amplitude Modulation- Frequency Modulation (AM-FM) models

- 2-D AM-FM Model
- Energy Separation Algorithm, Regularized Demodulation
- Dominant Component Analysis (DCA)

Filtering and modelling

- Model-based interpretation of Gabor filtering
- Alternative models for edge and smooth signals
- Texture / edge / smooth classification via model comparison

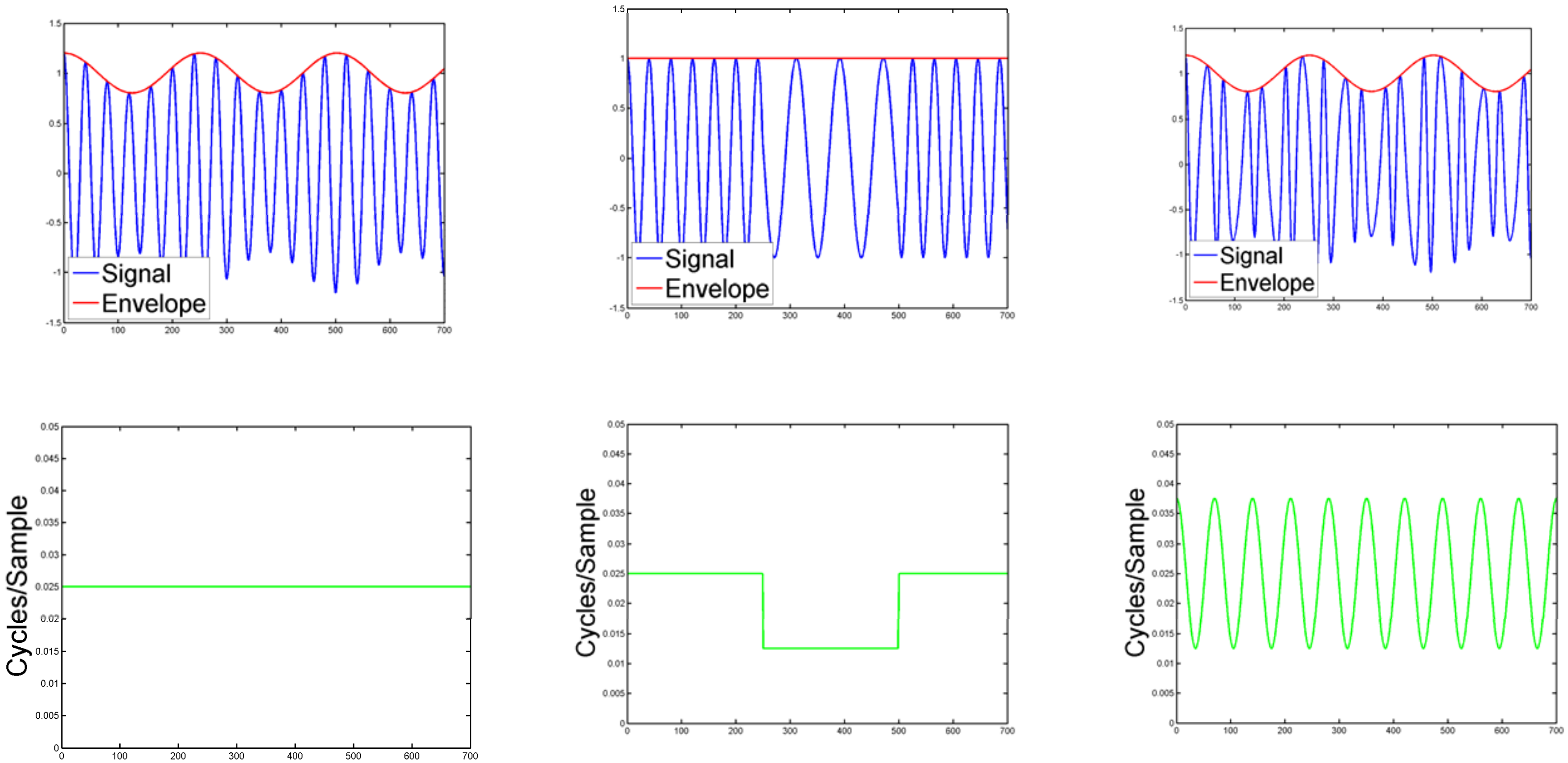
Applications to Segmentation

- Variational Image Segmentation using AM-FM features
- Weighted Curve Evolution for cue combination

1-D AM-FM Models

$$I(x) = a(x)\cos(\omega_c x + \int_0^x q(\tau)d\tau + \phi(0))$$

AM FM AM-FM



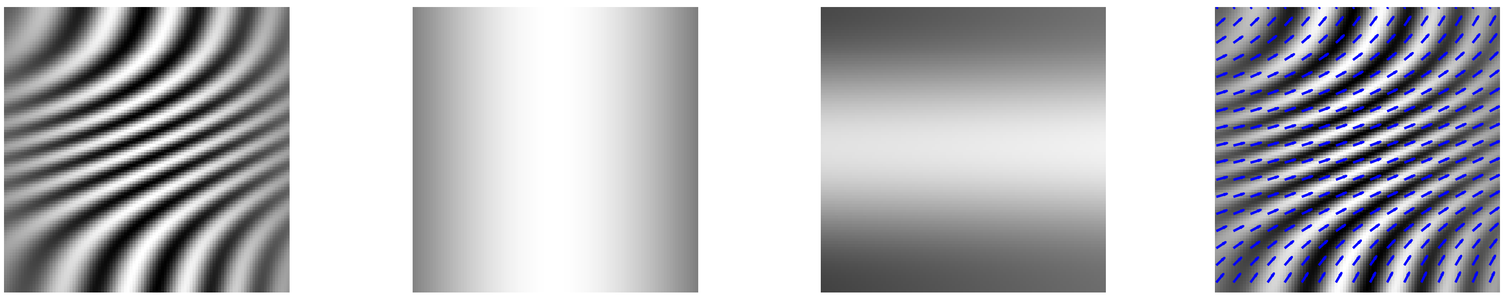
Applications: Telecommunications, Speech Analysis ...

2-D AM-FM models

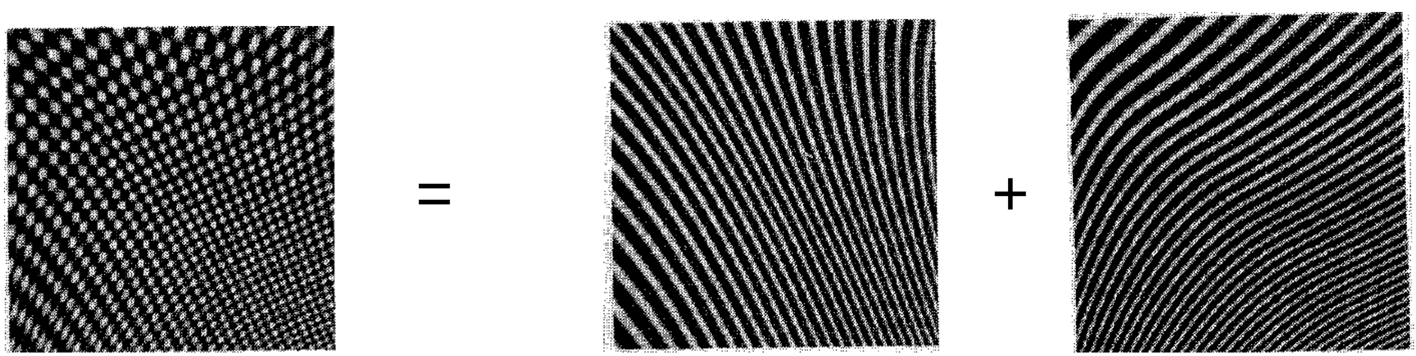
- Monocomponent AM-FM signal

$$I(x, y) = a(x, y)\cos(\phi(x, y)) + c, \quad \vec{\omega}(x, y) = \nabla\phi(x, y)$$

$I(x, y)$ $\quad$ $a(x, y)$ $\quad$ $|\vec{\omega}(x, y)|$ $\quad$ $\text{ang}(\vec{\omega}(x, y))$

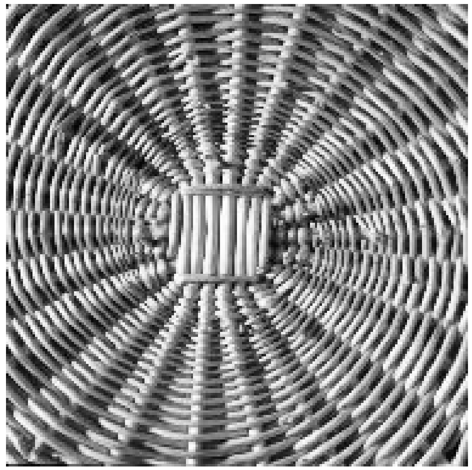


- Multicomponent AM-FM signals



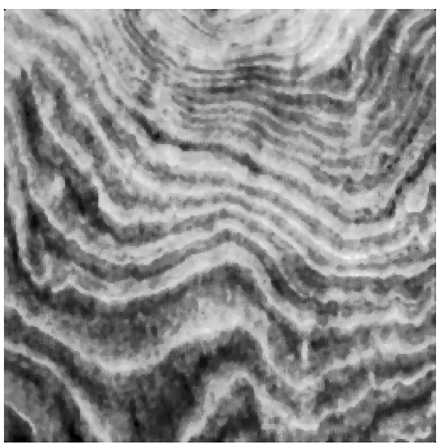
AM-FM models for Natural Images

- Man-made structures



The number of possible intensity images
notes the number of allowable gray levels
direct search, even for small ($m = 64$), t
Consequently, one is usually obliged t
assumptions about the image and degrad
as compromises at the computational st
putational problem is overcome by expl
servation that the posterior distribution i
approximately the same neighborhood
nal image, together with a sampling m
the *Gibbs Sampler*. Indeed, our princ
tribution is a general, practical, and mat
approach for investigating MRF's by sa
and by computing modes (Theorem

- Results of natural processes



AM-FM Demodulation: Energy Separation Algorithm

- Given, f recover $a, \nabla\phi$ s.t. $f(x,y) \simeq a(x,y)\cos(\phi(x,y))$
- Assume bandpass modulating signals
- Teager-Kaiser Energy Operator: $\Psi(I) \equiv ||\nabla I||^2 - \mathrm{tr}(\nabla^2 I)I$

$$\Psi[a\cos(\phi)] \simeq a^2|\vec{\omega}|^2$$

- Energy Separation Algorithm:

$$\frac{\Psi(f)}{\sqrt{\Psi(f_x)+\Psi(f_y)}} \approx |a(x,y)|$$

$$\sqrt{\frac{\Psi(f_x)}{\Psi(f)}} \approx |\omega_1(x,y)| \qquad \sqrt{\frac{\Psi(f_y)}{\Psi(f)}} \approx |\omega_2(x,y)|$$

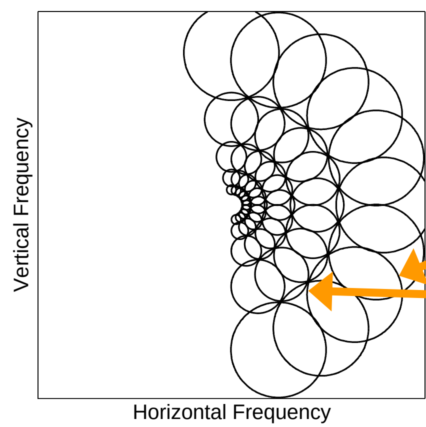
- Compared with Hilbert transform: locality

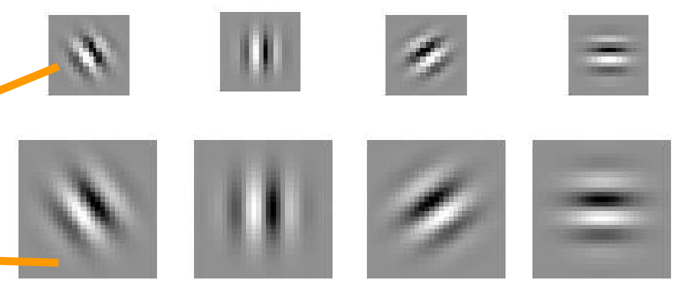
Refs. 1-D: Maragos, Quattieri & Kaiser, IEEE TSP ‘92, 2-D: Maragos & Bovik, JOSA ‘95

Natural Image Demodulation

- Problems:
  - Natural images do not satisfy ESA assumptions
  - Decomposition into AM-FM components: ill-posed problem
  - Effects of noise and approximations of derivatives
- Gabor filtering solution:
  - Break signal into simple components by Gabor filtering

Fourier transform isocurves



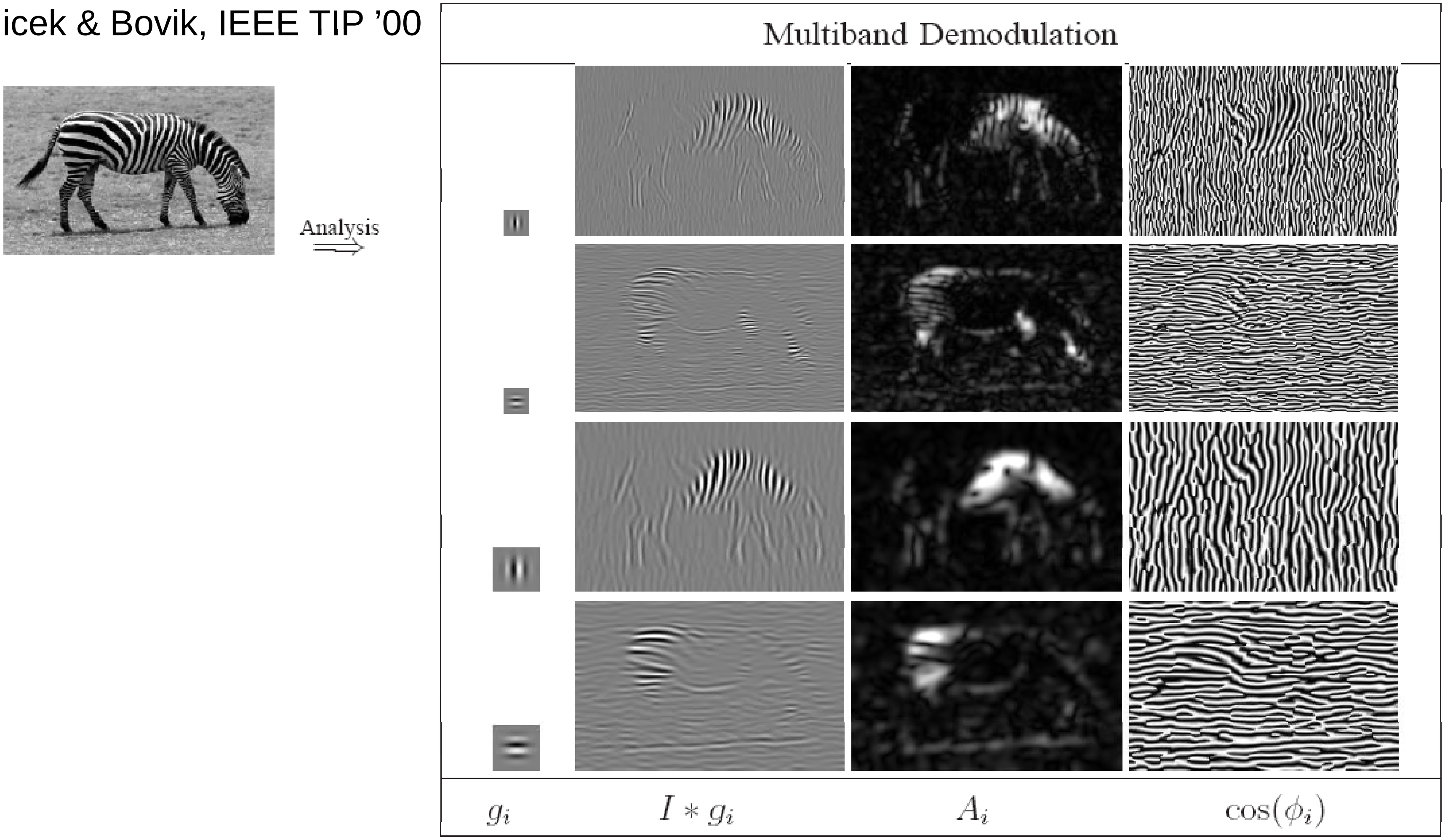


$$g(x, y) = \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right) \exp\left(j u_c x + j v_c y\right)$$

  - Demodulate individual outputs
  - Use derivative-of-Gabor filters to avoid differentiation

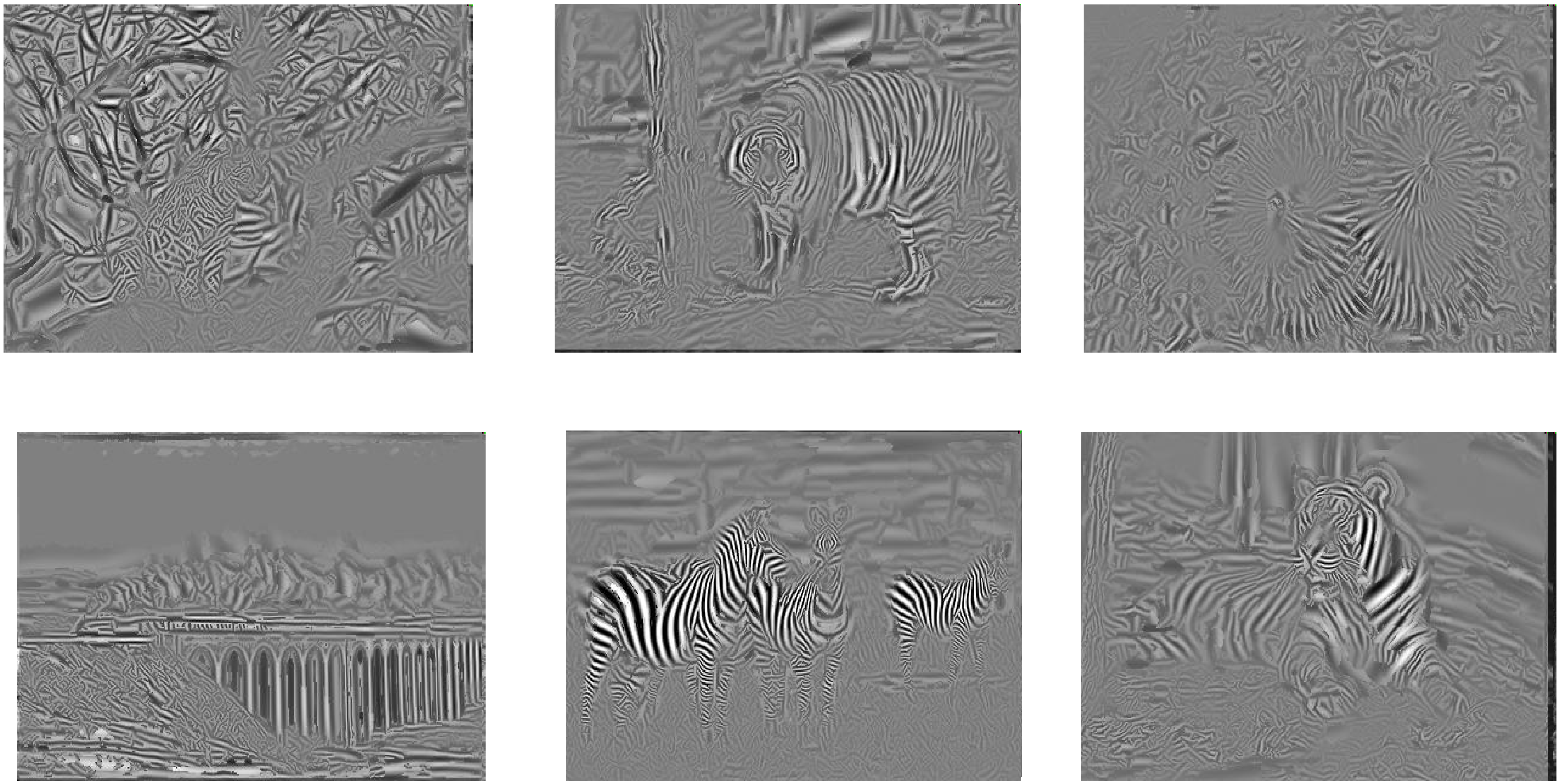
Channelized & Dominant Component Analysis

Havlicek & Bovik, IEEE TIP ’00



A_{DCA} ω_1 ω_2

DCA reconstruction of textured signals



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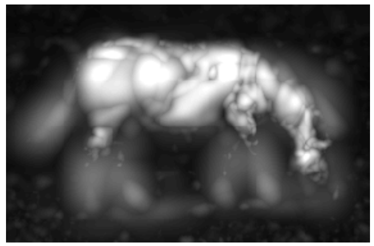
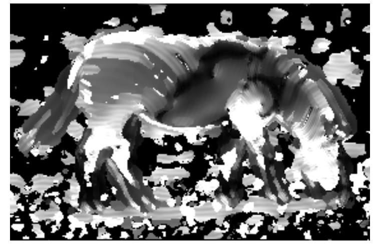
Motivation: deciding when to trust texture features

Input Image





DCA Features

- Model-based approach
  - Determine where the model fits the image well
  - Well = better than alternatives: Bayesian approach

- `Special treatment' for textured regions:
  - Fowlkes, Shi & Malik, Normalized Cuts for Segmentation
  - Meyer, Vese, Osher, U+V decomposition
  - Guo, Wu, Zhu, Texture + Sketch for reconstruction

Bayesian approach

- Synthesis model for each class

$$O(x) \simeq I_i(x|\mathcal{A}_i),$$

$O(x)$	Observations
$I_i(x)$	Synthesis of i-th model
$\mathcal{A}_i$	Parameters of i-th model

- Adopt probabilistic error model $P(O|\mathcal{A}_i, \mathcal{C}_i) = f(|O - I_i(\mathcal{A}_i)|)$
- Integrate out parameters to express observation likelihood given class

$$\begin{aligned} P(O|\mathcal{C}_i) &= \int_{\mathcal{A}_i} P(O|\mathcal{A}_i, \mathcal{C}_i) P(\mathcal{A}_i|\mathcal{C}_i) d\mathcal{A}_i \\ &\simeq P(O|\mathcal{A}_i^*, \mathcal{C}_i) P(\mathcal{A}_i^*|\mathcal{C}_i) \end{aligned}$$

- Derive class posterior using Bayes' rule

$$P(\mathcal{C}_i|O) = \frac{P(O|\mathcal{C}_i)P(\mathcal{C}_i)}{\sum_{k=1}^{K} P(O|\mathcal{C}_k)P(\mathcal{C}_k)} \simeq \frac{P_i(O|\mathcal{A}_i^*, \mathcal{C}_i)P_i(\mathcal{A}_i^*, \mathcal{C}_i)}{\sum_{k=1}^{K} P_k(O|\mathcal{A}_k^*, \mathcal{C}_i)P_k(\mathcal{A}_k^*|\mathcal{C}_i)}$$

Texture Model: sinusoid

- Model 1-D profile along principal orientation:

$$O(x) \simeq I_T(x; \{A, \phi, B\}) = A\cos(\omega x + \phi) + B$$

- Rewrite as expansion on linear basis:

$$I_T(x; \mathcal{A}) = \sum_{i=1}^{3} \mathcal{A}_i B_{T,i}(x)$$

$$\begin{aligned} \mathcal{A}_1 &= A\cos(\phi) & \qquad B_{T,1}(x) &= \cos(\omega x) \\ \mathcal{A}_2 &= -A\sin(\phi) & \qquad B_{T,2}(x) &= \sin(\omega x) \\ \mathcal{A}_3 &= B & \qquad B_{T,3}(x) &= 1 \end{aligned}$$

- Typical Matched filtering:
  - Project signal on sine/cosine basis (convolution with sine/cosine filters)
- Gabor filtering:
  - Filters have falloff (local analysis)

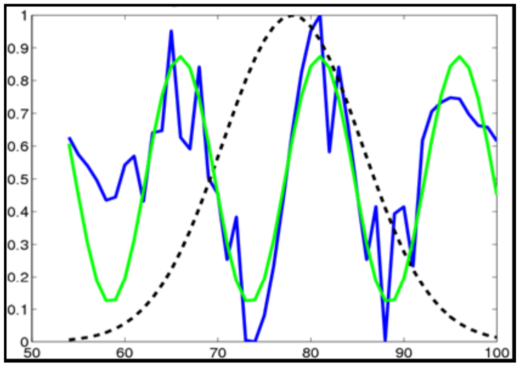
Probabilistic formulation of locality

- Leave distant data for a background model

$O(x)$ **observation at point x**

$I_i(x; \mathcal{A}_i)$ **model-based prediction**

$G(x)$ **probability that observation is due to foreground model**



$$P(O(x)|x, \mathcal{A}_T, \mathcal{C}_T) = \sum_{z_x=\{0,1\}} P(O(x), z_x|x, \mathcal{A}_T, \mathcal{C}_T)$$

$$= \sum_{z_x=\{0,1\}} P(O(x)|z_x, x, \mathcal{A}_T, \mathcal{C}_T) P(z_x|x)$$

$$= \underbrace{P(O(x)|x, \mathcal{A}_T, \mathcal{C}_T) G(x)}_{z_x=1} + \underbrace{P(O(x)|\mathcal{C}_B)(1 - G(x))}_{z_x=0}.$$

Lower bound of likelihood

- Likelihood for independent errors

$$\begin{aligned}\log P(O|\mathcal{A}) &= \sum_x \log P(O(x)|x,\mathcal{A}) = \\ &= \sum_x \log\left(G(x)P_f(O(x)|x,\mathcal{A}) + (1-G(x))P_b(O(x))\right) \\ &\geq \sum_x G(x)\log P_f(O(x)|x,\mathcal{A}) + \underbrace{(1-G(x))\log P_b(O(x))}_{c}\end{aligned}$$

- White Gaussian noise: weighted least squares

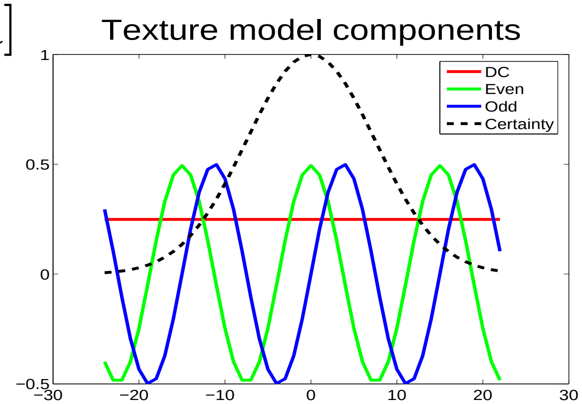
$$\mathcal{A}^* = \arg\max\left\{-\frac{1}{2\sigma^2}\sum_x G(x)[O(x)-I(x;\mathcal{A})]^2\right\} - \underbrace{\sum_x G(x)\log(\sqrt{2\pi}\sigma)}_{c'}$$

Gabor filtering as a weighted projection on a linear basis

- Rewrite lower bound in matrix form

$$\sum G(x)[O(x) - I(x;\mathcal{A})]^2 = [\mathcal{O} - \mathcal{B}\mathcal{A}]^T \mathcal{G} [\mathcal{O} - \mathcal{B}\mathcal{A}]$$

$$I_T(x;\mathcal{A}) = \sum_{i=1}^{3} \mathcal{A}_i B_{T,i}(x)$$



- Weighted least squares estimate

$$\mathcal{A}^* = D^{-1}\left(\mathcal{B}^T \mathcal{G} \mathcal{O}\right), \quad D = \mathcal{B}^T \mathcal{G} \mathcal{B}$$

- For diagonal D: parameters obtained by Gabor/Gaussian responses at $x = 0$

$$\mathcal{A}^*{}_1 = \sum_x G(x)\cos(x) O(x) \qquad \mathcal{A}^*{}_2 = \sum_x G(x)\sin(x) O(x) \qquad \mathcal{A}^*{}_3 = \sum_x G(x) O(x)$$

- Relation between Amplitude and bound

$$[\mathcal{O} - \mathcal{B}\mathcal{A}^*]^T \mathcal{G} [\mathcal{O} - \mathcal{B}\mathcal{A}^*] = \mathcal{O}^T \mathcal{G} \mathcal{O} - \mathcal{A}^{*T} D \mathcal{A}^*$$

$$\mathcal{A}^{*T} D \mathcal{A}^* = c\left[\frac{1}{2}(\mathcal{A}^{*2}_1 + \mathcal{A}^{*2}_2) + \mathcal{A}^{*2}_3\right] \propto \underbrace{A^2}_{\text{Local Amplitude}} + c'$$

$$\begin{aligned} \mathcal{A}_1 &= A\cos(\phi) \\ \mathcal{A}_2 &= -A\sin(\phi) \\ \mathcal{A}_3 &= B \end{aligned}$$

Alternative Hypotheses

- Cast edge detection in same setting:
  - Phase congruency model for edges & lines:

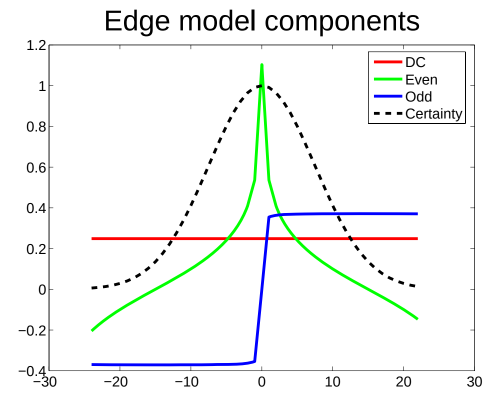
$$O(x) \simeq I_E(x) = A \sum_k a_k \cos(\omega_0 k x + \phi) + B$$

$$\phi = 0, \pi \quad \text{Line at } x = 0$$
$$\phi = \pm\frac{\pi}{2} \quad \text{Edge at } x = 0$$

  - Rewrite as expansion on basis:

$$I_E(x; \mathcal{A}) = \sum_{i=1}^{3} \mathcal{A}_i \mathcal{B}_{E,i}(x)$$

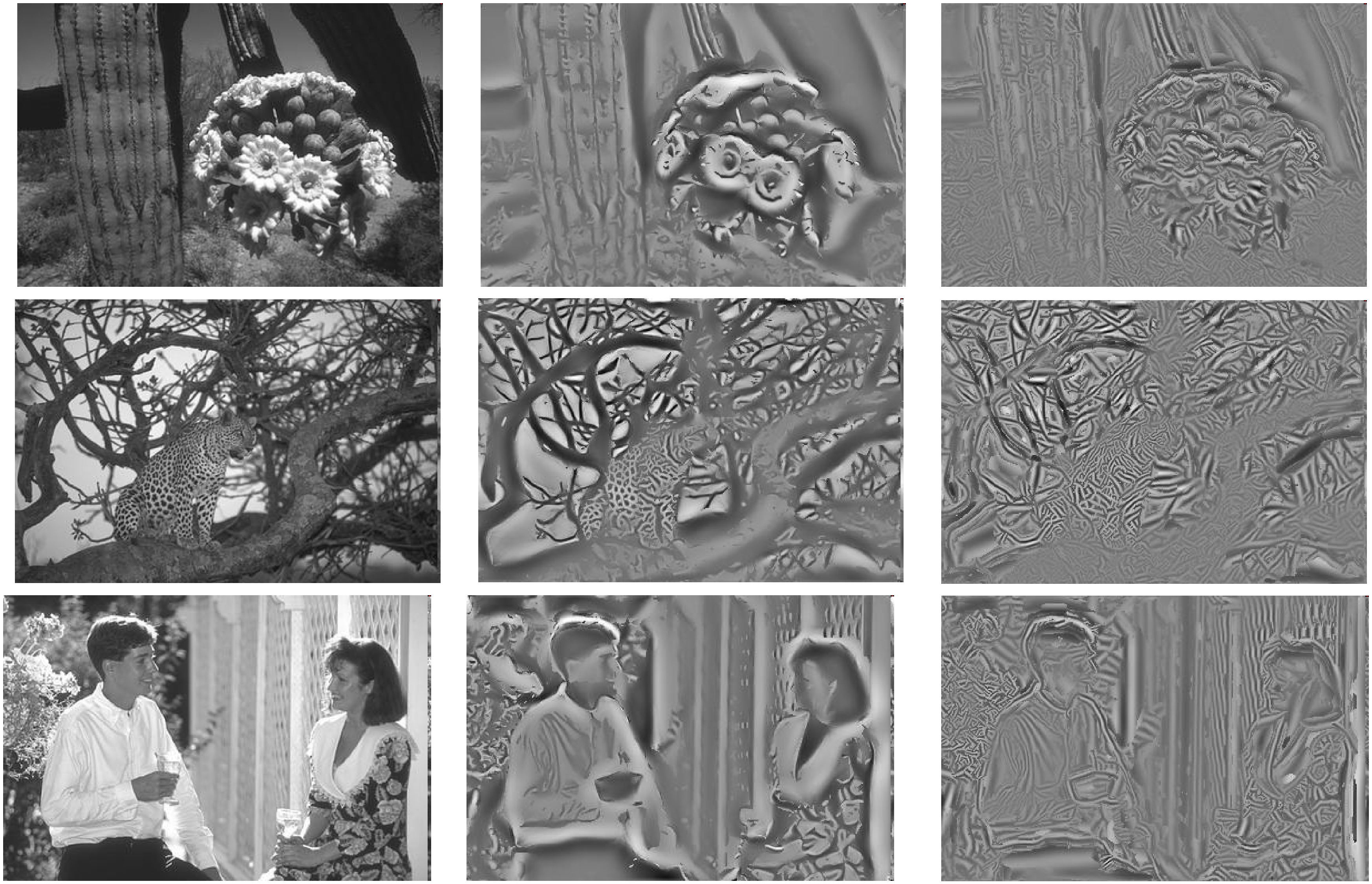
$$\begin{aligned} \mathcal{A}_1 &= A\cos(\phi) & \mathcal{B}_{E,1}(x) &= \sum a_k \cos(\omega_0 k x) \\ \mathcal{A}_2 &= -A\sin(\phi) & \mathcal{B}_{E,2}(x) &= \sum a_k \sin(\omega_0 k x) \\ \mathcal{A}_3 &= B & \mathcal{B}_{E,3}(x) &= 1 \end{aligned}$$



  - Iterate previous steps
  - Connection with Energy-based edge detection - QFPs
    - Morrone & Owens ‘87, Perona & Malik ’90,

- Smooth signal: $I_S(x) = B$

Structure captured by the Edge and Texture models



Input | Edge Reconstruction | Texture Reconstruction

Texture/Edge/Smooth discrimination in 2D images

- For each scale/orientation combination use all three models
  - Use Gabor/Edge/Gaussian filters to estimate model parameters

- Quantify gain of Edge/Texture hypothesis vs. Smooth hypothesis

$$\mathcal{G}_T = \log \frac{P(O|T)}{P(O|S)} = \frac{1}{2\sigma^2}\left(\mathcal{A}_T D_T \mathcal{A}_T^T - \mathcal{A}_S D_S \mathcal{A}_S^T\right) + c.$$

- Normalize for scale invariance: per-pixel gain

$$\mathcal{E}_T = \frac{\mathcal{G}_T}{\sum_x G_T(x)}$$

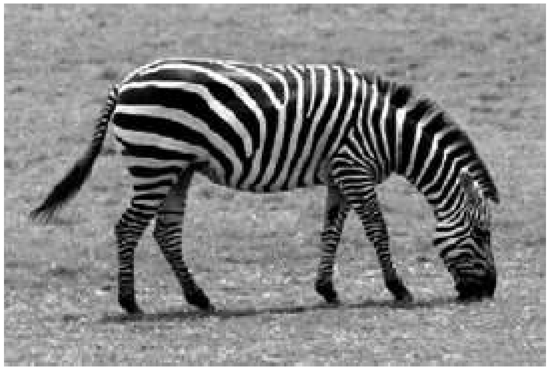
- Compute class posteriors

$$P(T|O) = \frac{P(O|T)}{P(O|T) + P(O|E) + P(O|S)} = \frac{R_T}{R_T + R_E + 1},$$

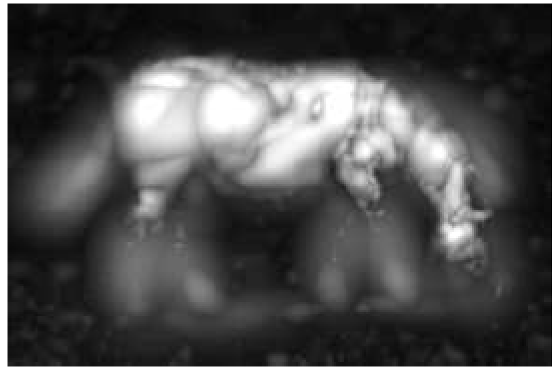
$$R_T = \frac{P(O|T)}{P(O|S)} = \frac{1}{1 + \exp\left(-\mathcal{E}_T\right)},$$

Text/Edge/Smooth Hypothesis Classification

Intensity



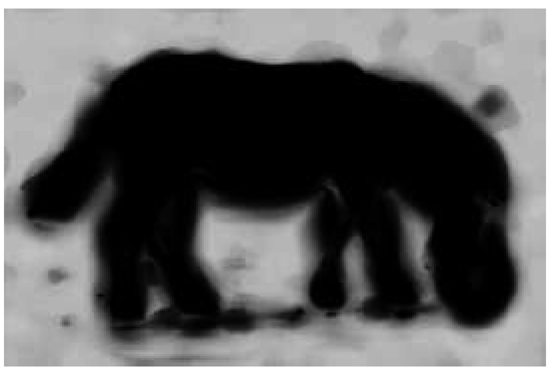
Texture Amplitude



Edge Amplitude

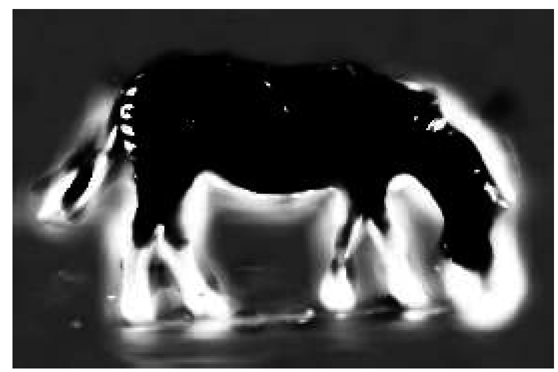
Posterior Probabilities

Prob(*Smooth*)

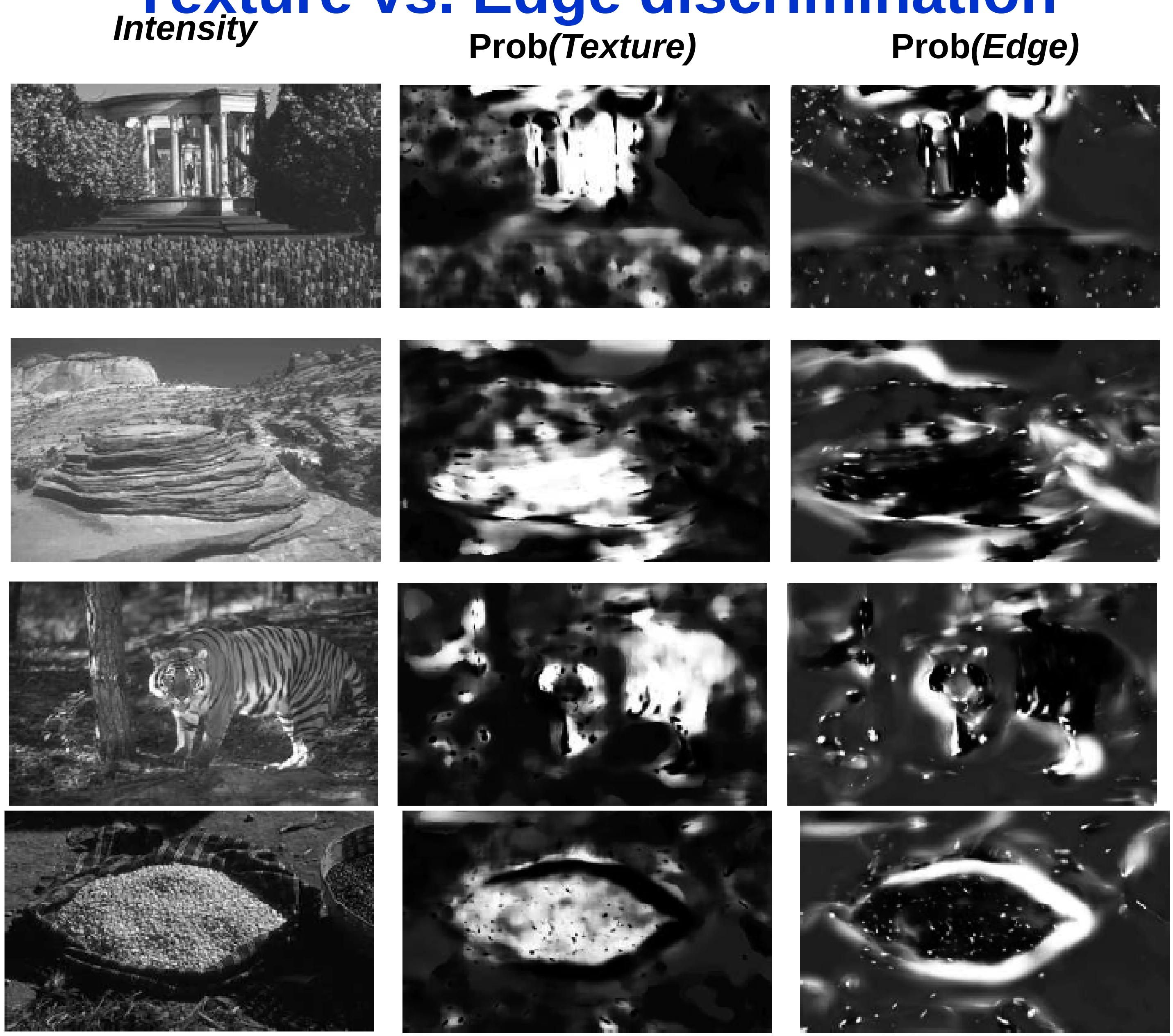


Prob(Texture)

Prob(Edge)



Texture vs. Edge discrimination



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Probabilistic Aspects

- Model-based interpretation of Gabor filtering
- Alternative models for edge and smooth signals
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Applications to Segmentation

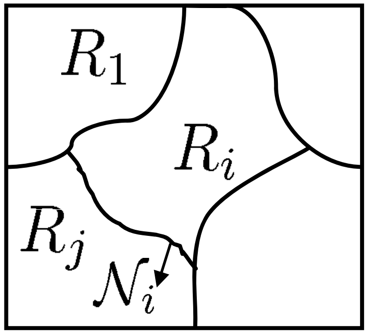
- Variational Image Segmentation using AM-FM features
- Weighted Curve Evolution for cue combination

Variational Image Segmentation

- Mumford & Shah '89
- Zhu & Yuille, '96: Region Competition Functional

$$J(\Gamma,\theta) = \sum_{i=1}^{M} \frac{\mu}{2}\int_{\Gamma_i} \mathrm{d}s - \iint_{R_i} \log P(I;\theta_i)$$

$$\frac{\partial \Gamma_i}{\partial t} = -\mu\kappa_i \mathcal{N}_i + \log\frac{P(I;\theta_i)}{P(I;\theta_j)}\mathcal{N}_i$$



- Level Set framework:
  - Chan & Vese, Scale-Space '99,
  - Yezzi, Chai & Willsky, ICCV '99
  - Paragios & Deriche, ICCV '99, ECCV '00
- Combination with Geodesic Active Contours (Paragios & Deriche):

$$\frac{\partial \Gamma_i}{\partial t} = \lambda \log\frac{P(I|\theta_i)}{P(I|\theta_j)}\mathcal{N}_i + (1-\lambda)\left[\nabla g \cdot \mathcal{N}_i - g\kappa_i\right]\mathcal{N}_i$$

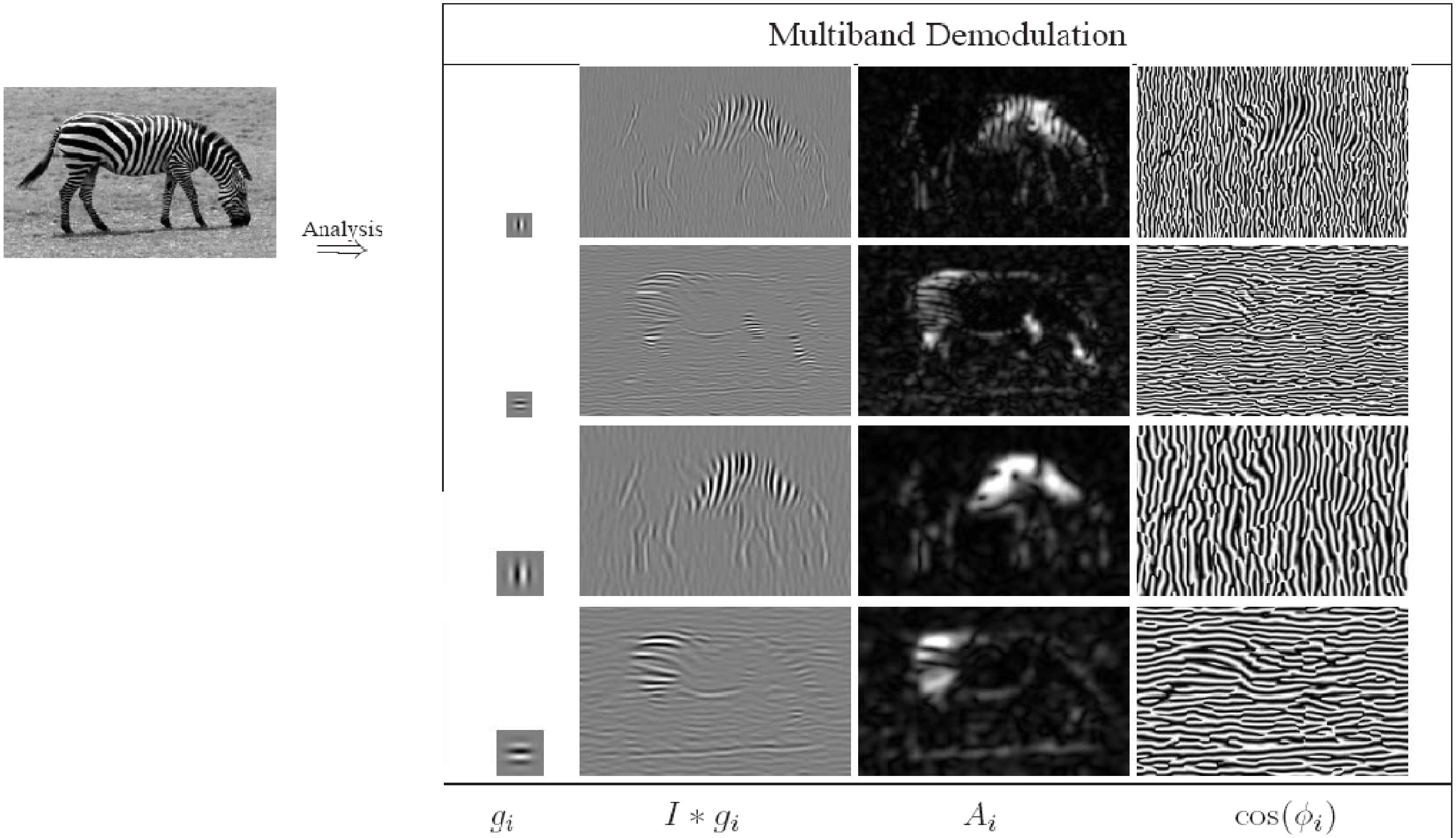
Features for Variational Texture Segmentation

- Filterbank-based methods
  - Zhu & Yuille, PAMI '96: Small filterbank, few results on texture
  - Paragios & Deriche, IJCV '02: Supervised
  - Sagiv, Sochen et al. , '02. Sandbert Chan & Vese et al, '02 : Feature selection

- Histograms
  - Kim, Fisher & Willsky, ICIP `01: Nonparametric estimate of intensity
  - Tu & Zhu, PAMI '02: Histograms of intensity + model calibration

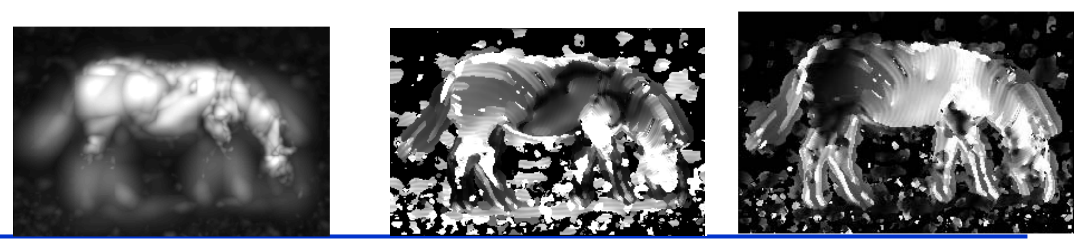
- Low dimensional descriptors
  - Zray, Havlicek, Acton & Pattichis, ICIP '01: Modulation features + clustering
  - Vese & Osher, JSC '02, g_1, g_2 features from $u + v$ decomposition
  - Rousson, Brox & Deriche, CVPR '03: Anisotropic diffusion + structure tensor.

Modulation features via Dominant Component Analysis



DCA

$$
\begin{aligned}
i(x,y) &= \arg \max_{1 \le k \le K} A_k(x,y), \\
A_{\text{DCA}}(x,y) &= A_{i(x,y)}(x,y), \\
\vec{\omega}_{\text{DCA}}(x,y) &= \vec{\omega}_{i(x,y)}(x,y)
\end{aligned}
$$

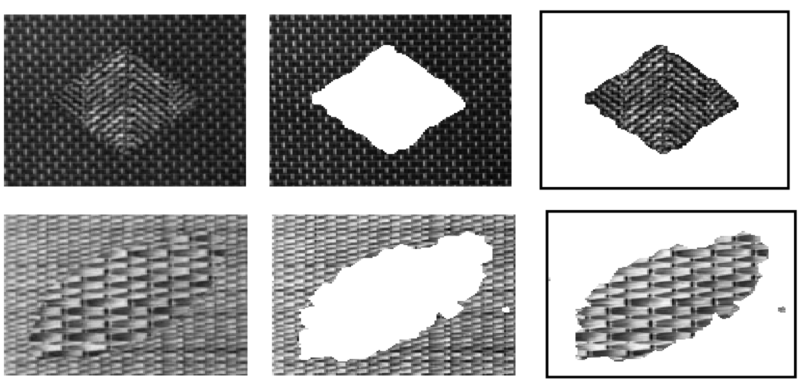


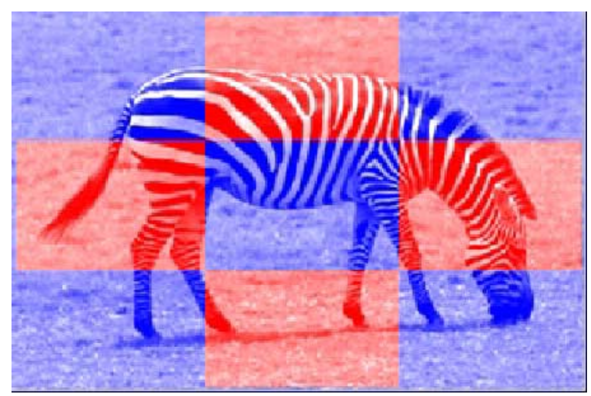
A_{DCA} ω_1 ω_2

Variational Segmentation with Modulation Features

- 3-dimensional feature vector
  - Amplitude function: $a(x, y)$ Contrast
  - Magnitude of frequency vector: $|\vec{\omega}(x)|$ Scale
  - Angle of frequency vector: $\angle\vec{\omega}(x)$ Orientation
- Smooth, low-dimensional descriptor
- Gaussian distribution for $a(x, y)\,|\vec{\omega}(x)|$, von-Mises for $\angle\vec{\omega}(x)$

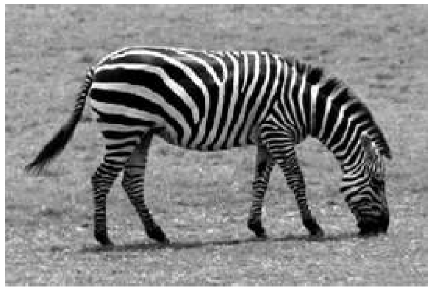
- Initialize segmentation randomly and iterate:
  - Estimate region parameters using current segmentation
  - Modify segmentation by curve evolution



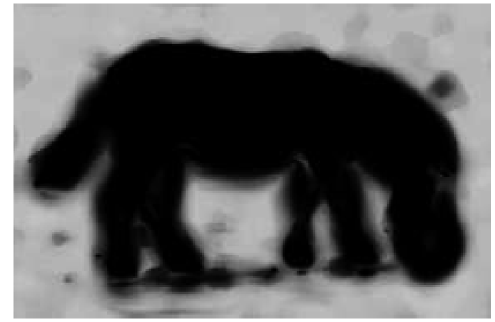


Cue Combination Task

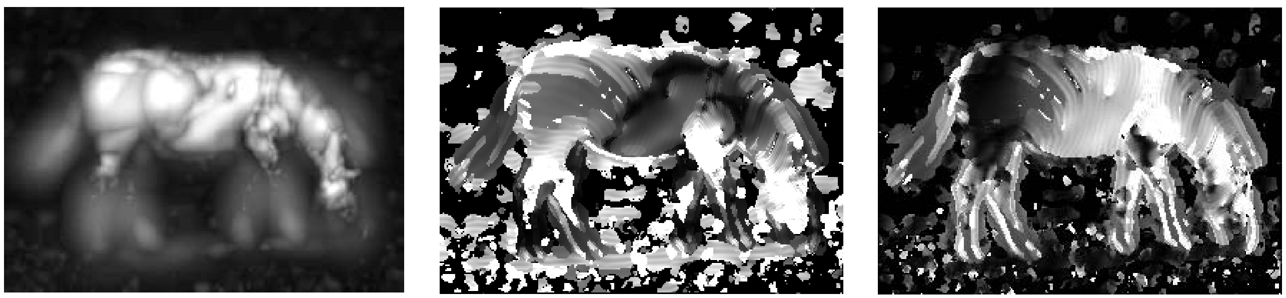
Intensity



Prob(*Smooth*)



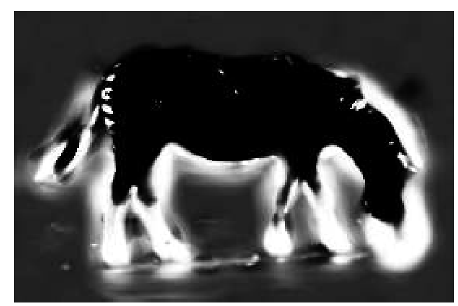
Texture Features



Prob(Texture)

Edge Strength

Prob(Edge)



Classifier Combination Approach

- Treat probabilistic balloon force of RC as log-odds of two-class classifier

$$\mathcal{L}_F = \log \frac{P(F; a_i)}{P(F; a_j)}$$

  - Decide about pixel label by comparing feature likelihoods
- Consider separate classifiers based on texture/intensity/edge cues

- \`Supra –Bayesian' classifier combination, a.k.a. \`stacking'
  - Treat classifier outputs themselves as random variables

$$P(\mathcal{L}_F|i) \propto N(\mu_i, \sigma^2) \qquad P(\mathcal{L}_F|j) \propto N(\mu_j, \sigma^2),$$

  - Ideally, $\mu_j \ll 0, \mu_i \gg 0, \sigma \ll 1$
  - Consider joint distribution of vector of classifier log-odds.
  - For independent classifiers s.t. $\mu_j = -\mu_i$ decision is given by

$$\log \frac{P(i; \mathcal{L})}{P(j; \mathcal{L})} = \sum_k \frac{\mathcal{L}_k}{\sigma_k^2}$$

Weighted Curve Evolution

- Last slide summary: give higher weight to log-odds of better classifier

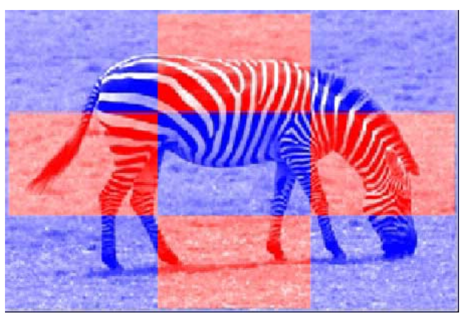
$$\log \frac{P(i;\mathcal{L})}{P(j;\mathcal{L})} = \sum_k \frac{\mathcal{L}_k}{\sigma_k^2}$$

- Adaptation to curve evolution: set weights equal to class posteriors
- Weighted curve evolution:

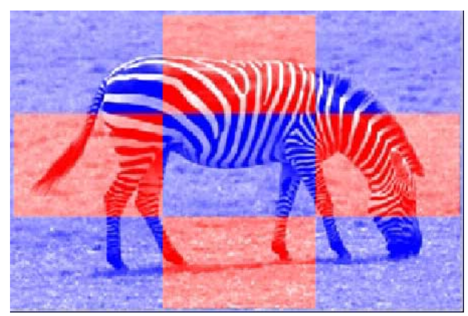
$$\frac{\partial \Gamma_i}{\partial t} = \left[w_T \log \frac{P(F_T;\alpha_{T,i})}{P(F_T;\alpha_{T,j})} + w_S \log \frac{P(F_S;\alpha_{S,i})}{P(F_S;\alpha_{S,j})} + w_E \left[(\nabla g \cdot \mathcal{N}_i) - g\kappa \right] \right] \mathcal{N}_i.$$

- Compare to Geodesic Active Regions

$$\frac{\partial \Gamma_i}{\partial t} = \lambda \log \frac{P(I|\theta_i)}{P(I|\theta_j)} \mathcal{N}_i + (1-\lambda) \left[\nabla g \cdot \mathcal{N}_i - g\kappa_i \right] \mathcal{N}_i$$

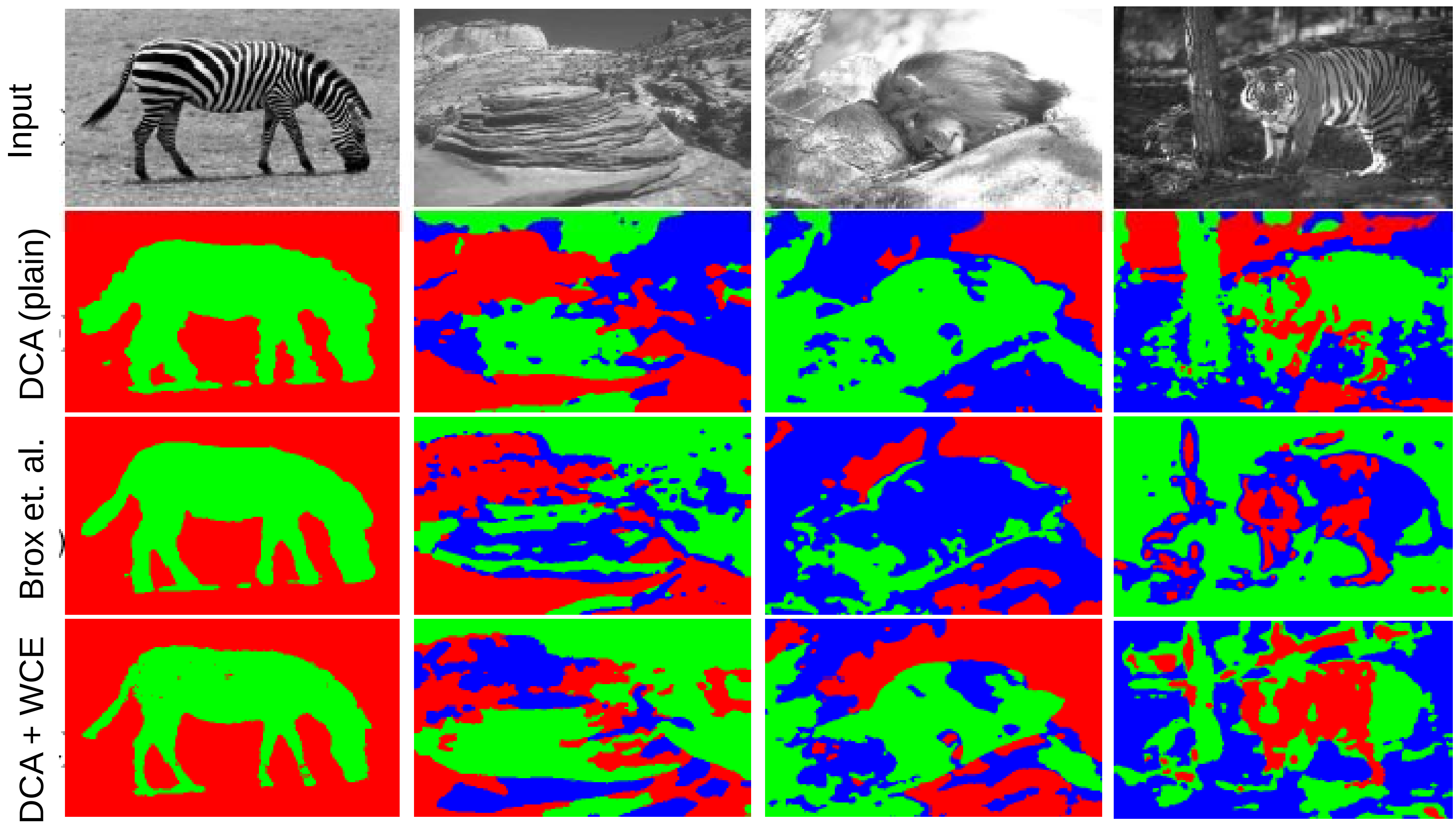


Geodesic Active Regions



Weighted Curve Evolution

Segmentation Result Comparisons



Quantitative Evaluation

- Berkeley Benchmark: 100 hand-segmented images (test-set)
- Bidirectional Consistency Error
  - At each pixel: normalized set difference of machine- and user- regions

$$E(S_M, S_k, p_i) = \frac{|R_M(p_i) \setminus R_k(p_i)|}{|R_M(p_i)|}$$

  - Make symmetric, take minimum over users, and average

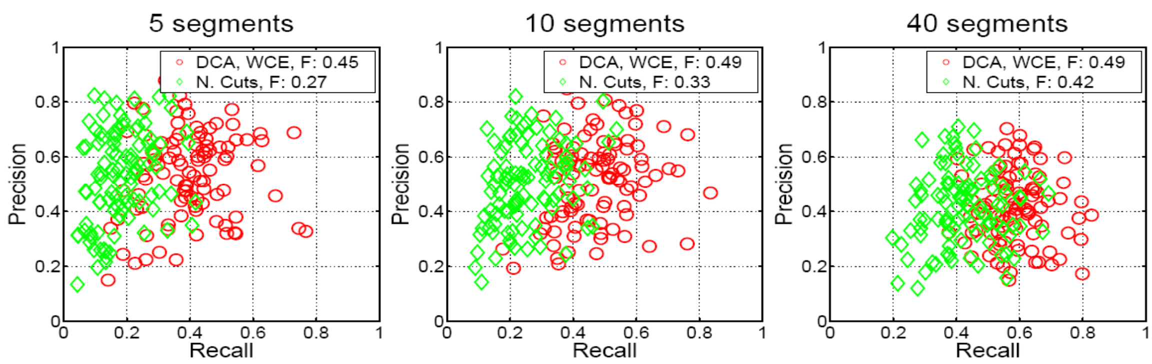
$$BCE(S_M) = \frac{1}{n}\sum_{i=1}^{n} \min_k \max(E(S_M, S_k, p_i), E(S_k, S_M, p_i))$$

Fronts	Optimal	2	3	4	5	6	7	8	9	10
DCA, WCE	.38/.39	.46/.49	.49/.51	.51/.52	.54/.53	.54/.53	.57/.57	.59/.58	.59/.59	.61/.61
DCA, Plain	.39/.39	.48/.51	.50/.51	.51/.52	.54/.53	.56/.56	.58/.58	.60/.59	.62/.61	.63/.61
[49], WCE	.40/.41	.47/.49	.51/.53	.52/.53	.55/.55	.57/.58	.59/.58	.60/.60	.62/.61	.63/.63
[49], Plain	.40/.42	.48/.50	.52/.53	.55/.54	.56/.56	.59/.59	.60/.60	.63/.62	.63/.63	.65/.65
N. Cuts	.41/.43	.49/.51	.52/.53	.55/.53	.55/.55	.59/.60	.60/.59	.63/.61	.63/.63	.64/.65

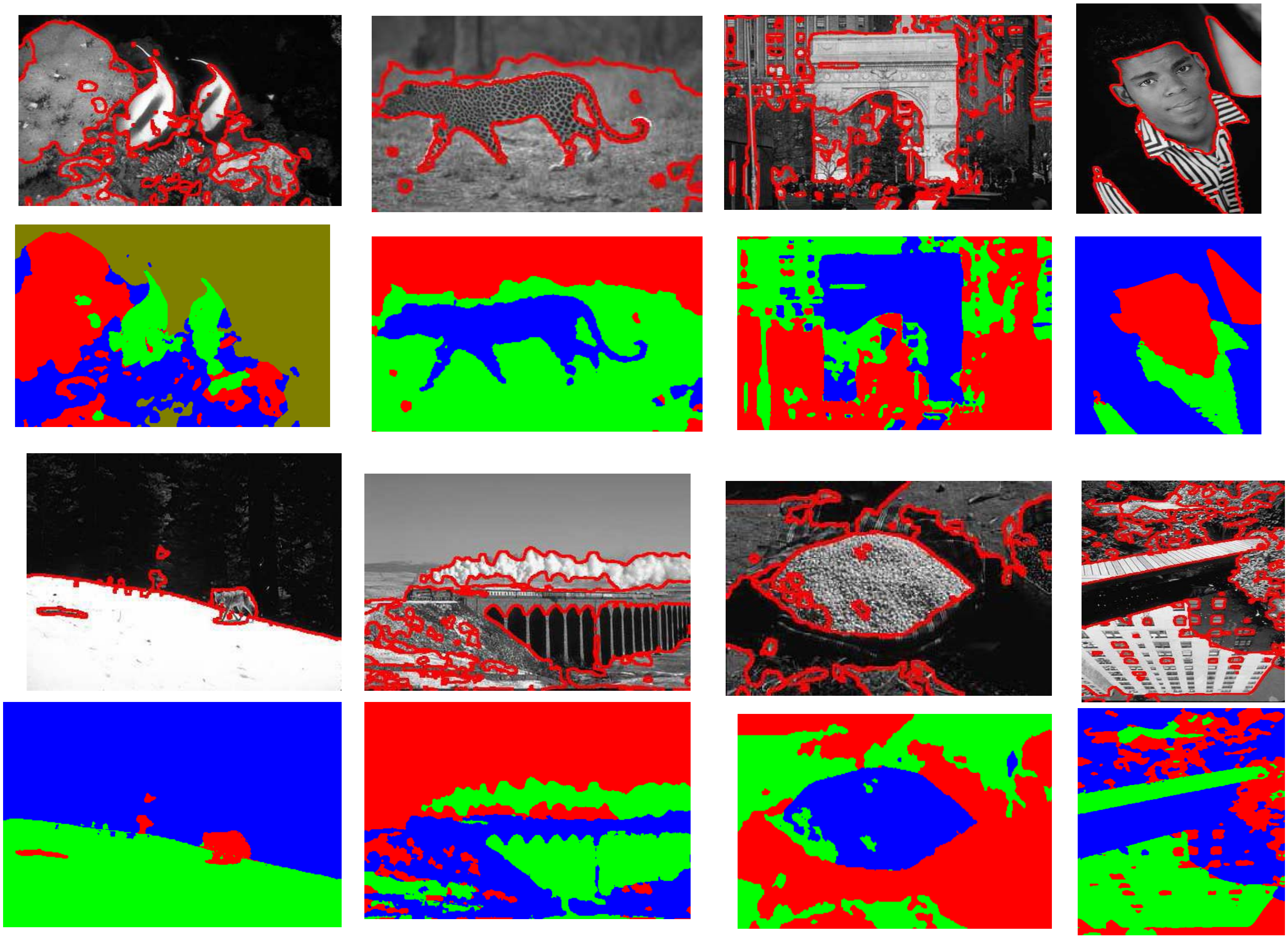
- Precision-Recall

$$\text{Precision} = \frac{\#\text{True}}{\#\text{True} + \text{False}}$$

$$\text{Recall} = \frac{\#\text{True}}{\#\text{Actual}}$$



Berkeley Dataset Segmentations



Conclusions & Future Work

- AM FM models: naturally suited for modelling oscillations
  - Efficient and reliable parameter estimation
  - Low-dimensional descriptors
- Model-based interpretation of feature extraction
  - Gabor filtering
  - Energy-based feature detection
- Cue Combination for Curve Evolution

- Future work
  - AM FM models: synthesis, PDE methods (G. Evangelopoulos)
  - Integrate with other structures
    - Crosses, junctions, blobs, ridges
  - Use segmentation to drive object detection
    - Use segments as elementary image structures
    - Construct segment-based object representations

Synthetic signal reconstruction